REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From – To)
November 2001	FINAL	December 2000 – July 2001

4. TITLE AND SUBTITLE
Applying Optimization to Marine Corps Decision Making for Repairable Item Lateral Redistribution Policy

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
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5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
CMC (LX)
HQMC DC I&L
2 Navy Annex
Washington, D.C. 20380-1775

8. PERFORMING ORGANIZATOIN REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Same as performing organization above

10. SPONSOR/MONITOR'S ACRONYM(S)
CMC (LX)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
The views expressed in this paper are those of the authors and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

14. ABSTRACT

We demonstrate some of the immediate cost benefits of centrally managing a multi-site inventory of common repairable parts originally managed as separate stocks. First, we modify the basic transportation problem to determine the lowest global cost for redistributing parts between the multiple stockage points. Our initial solution results in a $3.7 million reduction in purchasing costs. Next, we show that by developing a consolidated shipments model, we are able to reduce the cost to fill demand, with available stock, by 20%. We also highlight how we determine per unit and consolidated shipment costs, and essential data elements for this type of model. Finally, we conduct sensitivity analysis on the model output to show how a central stock manager can make tradeoff decisions between cost and readiness.

15. SUBJECT TERMS
Mixed Integer Programming, Transportation Problem, Optimization, Consolidated Shipments, Lateral Redistribution, Repairable Parts Stockage

16. SECURITY CLASSIFCATION:
UNCLASSIFIED

b. REPORT	b. ABSTRACT	c. THIS PAGE
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED

17. LIMITATION OF ABSTRACT
None

18. NUMBER OF PAGES
12

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Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Sta. 239-18
298-102

20020904 060

Applying Optimization to Improve Marine Corps Decision Making for Repairable Item Lateral Redistribution Policy

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Abstract

We demonstrate some of the immediate cost benefits of centrally managing a multi-site inventory of common repairable parts originally managed as separate stocks. First, we modify the basic transportation problem to determine the lowest global cost for redistributing parts between the multiple stockage points. Our initial solution results in a $3.7 million reduction in purchasing costs. Next, we show that by developing a consolidated shipments model, we are able to reduce the cost to fill demand, with available stock, by 20%. We also highlight how we determine per unit and consolidated shipment costs, and essential data elements for this type of model. Finally, we conduct sensitivity analysis on the model output to show how a central stock manager can make tradeoff decisions between cost and readiness.

Introduction

The Marine Corps logistics community is continually focused on improving support to our operating forces, while reducing both deployment "footprint" and support costs. The Marine Corps holds repairable parts stock at six separate locations (Figure 1). Each location has traditionally managed their stocks independently, by making stockage determinations and purchase decisions at different times to support
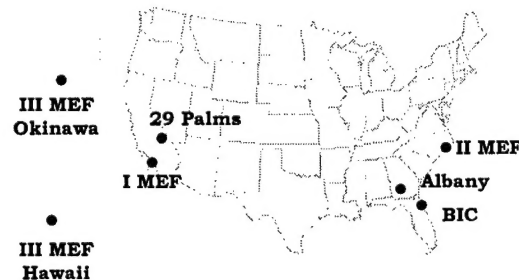
Figure 1. The Marine Corps manages repairable parts stocks at these six geographically separate locations.

its specific customer base. Each site uses a demand-based approach to determine stockage requirements on a periodic basis. We expect that by combining distributed demands from multiple customer bases and by centrally managing the separate stockage sites, the global inventory system will improve customer service and reduce total supply system costs. Several references, including (Kukreja et. al., 2001), show the benefits of consolidating stock management including using transshipments to fill stock requirements. As part of the effort to centralize repairable parts management, the Marine Corps consolidated all of the individual data domains into a central repository. This results in more accurate accountability data and the ability to determine stockage requirements for each site simultaneously. As a result, parts excesses and deficiencies were identified for

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each site. Since each site has managed their separate inventory for several years, large amounts of excesses and deficiencies were identified across the global inventory. Now, a central manager is able to make decisions to either laterally redistribute excess parts or to purchase them from a supply source in order to fill all deficiencies.

We develop our optimization models to assist decision makers by determining a plan with the lowest total cost to fill all parts deficiencies. Our effort is focused on the result of this initial stockage review. We fill demand at each site by either purchasing from a supply source or laterally redistributing from another excess site. The plan for meeting global demand depends on freight shipping cost between sites, the unit price of each commodity, and the amount of the commodity available from all excess sites. We use our model after the periodic stockage review to determine the best plan for filling demand. Once the deficiencies are filled, continuous purchase and redistribution decisions can be made for all sites on a daily basis, without the use of our model.

Related Research

The operations research community is rich with references on the basic transportation problem and the benefits of using consolidated shipments for reducing total transportation costs. We explore several of them and summarize some of their applicable findings here. Many references also describe the history of the basic transportation problem. For example, (G. Thompson, 1992) describes the origin of the *transportation problem*, its initial use, and applications, specifically related to economic modeling. We used the example formulation that Dr. R. Rosenthal implements in his tutorial (GAMS, 1997) as

the basis for our first model. Dr. Rosenthal's use of the transportation problem, an example for how the algebraic formulas in GAMS models scale to large data sets, applies directly to how we adapt the model to our problem. (Kukreja, et. al., 2001) compares a decentralized inventory control policy for independent stockage sites to one that is centralized. Although based on a continuous review inventory policy, they demonstrate how using transshipments between sites to fill demands can significantly reduce total inventory costs. (Geoffrion and Powers, 1981) describe how management support systems are used in the context of a manager's decision cycle. They discuss the four necessary components: data files, the model, the solver, and the decision maker interface. Furthermore, their discussion highlighting the tradeoffs between each component and modeling complexity provided useful insight into our model development efforts. Their observation of finding the right compromise between data requirements, modeling realism, and solvability is especially applicable to our problem. Geoffrion's earlier article (Geoffrion, A.M., 1976) concerning distribution planning also provides a summary of modeling facts and assumptions for distribution models that remain relevant today. In their 1994 article, (Bausch, et. al., 1994) describes an optimization model that provides minimum cost dispatches where multiple shipping modes are available. They discuss the importance of using cost as the principal measure of effectiveness even though consolidated transportation costs are typically neither linear nor continuous. This provides motivation for how we compute our consolidated transportation cost estimates and model the resulting non-linear, discontinuous function. Finally, (Brown and Ronen, 1997) describe how they developed and implemented an order consolidation

system at a major US manufacturer, saving hours of the users' time usually spent determining the schedule using manual methods. They also show a significant transportation cost savings of 5% which translates to about $1.7 million.

Problem Scope

For our problem, we identify 323 commodity types that were in both excess and deficient status at six different sites. There are 2581 items available to satisfy a total demand of 1986 deficiencies. Many (983) of the deficiencies do not have associated excesses available and have purchased at a cost of $4.8 million. Our models do not attempt to avoid this cost. For the 1003 deficiencies that could be filled through redistribution from an excess site, the model chooses to either buy from a supply source or laterally redistribute from a site in an excess posture. All 1003 deficiencies could be purchased at a cost of $3.86 million. This is the cost we set out to minimize through redistribution and purchases. Without redistribution, all deficiencies could be filled at a total purchase cost of $8.66 million.

Heuristic Approach

The Marine Corps' current supply information system uses a heuristic to approach this problem and provide a redistribution plan. The heuristic is a greedy assignment algorithm that uses the geographical distance between inventory sites to assign redistributions between excess and deficient sites for each item. The program makes assignment decisions by examining excess locations sequentially, for each deficient site, once during each cycle run. One cycle is run each day. The algorithm iterates through each deficient location, inspects the closest site for an

excess item, thereby determining if it can fill the deficiency. If the site has an excess item, the deficiency is decremented by the appropriate amount. Otherwise, another site is inspected during the next cycle. This process has several limitations. First, it takes up to six days to cycle through all the sites and determine the resulting plan. This adds at least six days to the shipping lead time just to cycle through the program. Second, the program makes no distinction between units located in Hawaii and those located in Okinawa. Therefore, it doesn't consider shipments between the locations and doesn't identify any difference in cost between shipping to or from them. Finally, due to its design, the heuristic does not require that all deficiencies be filled from all available excesses before exiting the algorithm. Based on these deficiencies, we do not compare its output to each of our models. Therefore, we do not attempt to reproduce its results. We know that optimization could help make better redistribution and purchasing decisions and set out to solve this problem using modifications to the basic transportation problem.
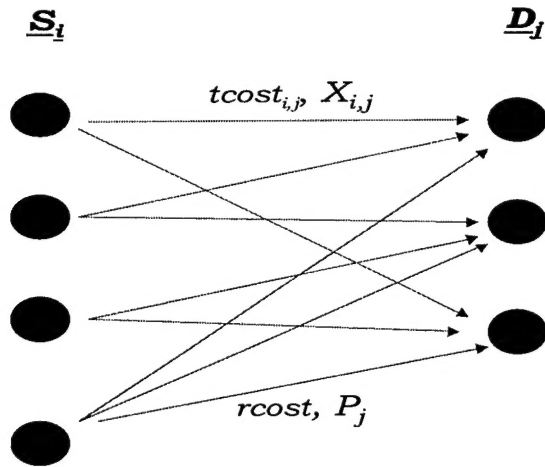
Initial Redistribution Model

We extend the basic transportation problem (GAMS, 1997) by adding a slack variable representing the amount of each commodity purchased at the associated unit price. We use this source-dependent cost to ensure all demand is satisfied at the deficient sites that lack enough supply from the excess sites.

Model

For each commodity deficiency, this model chooses the cheapest source based on excesses, demand, purchase price, and shipping cost associated with each feasible arc. Figure 2 shows a graphical

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representation of this model. Appendix 1 provides a detailed formulation and explanation.


Figure 2. A graphical representation of the initial redistribution problem for one of the 323 commodities. Each commodity has a set of supply and demand nodes. Each supply node, S_i, represents a site with an excess quantity of the commodity. Demand nodes, D_j, represent sites with one or more deficient quantities. Each arc includes assignment variables, $X_{i,j}$, with the associated per unit shipping costs, $tcost_{i,j}$. Each commodity includes a dummy supply node with arcs connecting it to each demand node. These arcs include a slack variable, P_j, representing the amount purchased and the associated purchase cost penalty rcost. P_j is uncapacitated.

Determining per unit shipment costs

Determining shipment costs for each arc is the most difficult and time consuming step in developing this initial model. However, we feel that the fidelity of our model relies on our ability to accurately estimate these costs. Our unit shipment costs are based on item weight, mode of shipment, and origin-destination pairs. We consider several shipment modes including truck load (TL), less than truckload (LTL), Air Mobility Command (AMC) military aircraft, the United States Postal Service (USPS), DHL Worldwide Express, and Federal Express (FEDEX) options. Delivery times for all these modes are acceptable. We predetermine the mode each item will be shipped based on the minimum cost between each origin and destination pair and specific weight criteria. These modes are shown in figure 3. TL, LTL, and AMC shipment costs are based on a spreadsheet model provided by the Freight Routing Division of the Deployment Support Command of the Military Traffic Management Command (MTMC). LTL costs are based on the most recent Roadway Express tender. TL costs consider the top five rates between origin and destination pairs and are based on the mileage and shipment weight. AMC, international air rates, are used for all shipments between Hawaii and Okinawa, shipments over 150 lbs between Okinawa and CONUS units, and for shipments over
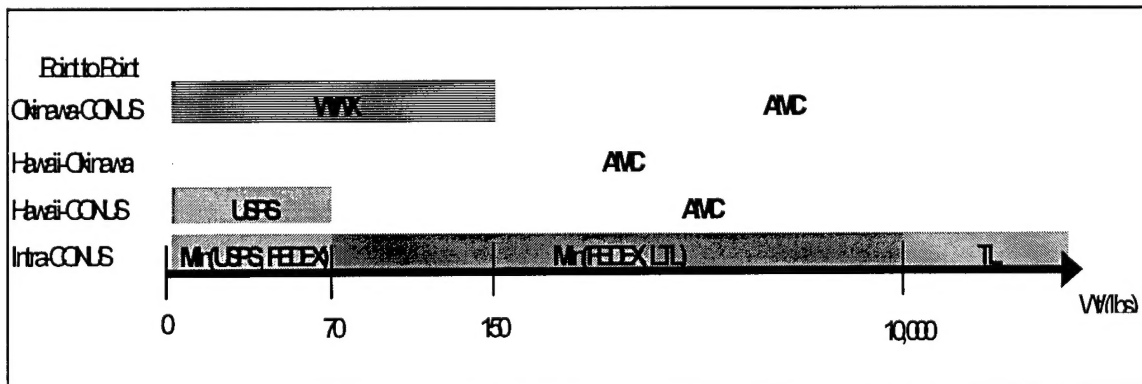
Figure 3. We selected the shipment modes between each origin – destination pair by selecting the one with the lowest cost. This shows how the modes were selected based on each commodity's unit weight.

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and destination pairs and are based on the mileage and shipment weight. AMC, international air rates, are used for all shipments between Hawaii and Okinawa, shipments over 150 lbs between Okinawa and CONUS units, and for shipments over 70 lbs between Hawaii and CONUS units. Surface rates are not considered for overseas shipments due to longer shipping times. We use a linear regression of postal rates from United States Postal Service (USPS) data for shipments under 70 lbs for both Hawaii to CONUS and intra-CONUS shipments. We also consider Federal Express (FEDEX) for intra-CONUS shipments under 70 lbs. We only use FEDEX if shipment costs are less that the USPS rate. We use rates from the World Wide Express contract for Okinawa and CONUS shipments weighing under 150 lbs.

Implementation

Our GAMS model generates: 5121 equations, 58,234 non-zero variables, and 22401 single variables. The model solved for an optimal solution in 0.71 seconds using the XA solver (GAMS, 1997) on a Dell 800 MHz personal computer.

Model output summary

Our initial redistribution model recommends filling all deficiencies with items from excess sites at a shipping cost of approximately $80 thousand. Adding this to the required purchase cost, the total cost to fill all deficiencies becomes approximately $4.8 million. Figure 4 compares the initial requirement for purchasing all deficiencies under an individual site management policy to the costs recommended using our intitial redistribution model. By not purchasing those items that could be redistributed from excess sites, the plan from our initial model

allows the Marine Corps to avoid spending approximately $3.8 million in purchases.
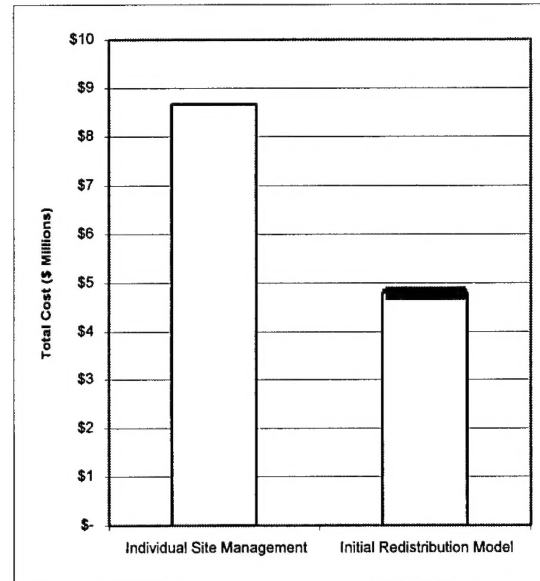
Figure 4. Comparing the total requirement to purchase all repairable part deficiencies under an independent site management plan to the results provided by our initial redistribution model. This represents a 45% reduction in total cost.

Consolidated Shipments Model

The redistribution plan generated by our Initial Redistribution Model results in a 45% savings in total cost relative to the cost of independent site management. Assigning the lowest cost mode to by item shipments lowers the total transportation costs between each origin and destination pair for each commodity. However, based on research of several consolidated shipments models, specifically (Bausch, et. al., 1994) and (Brown, Ronen, 1997), we expect that consolidating shipments between sites allow even greater savings.

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Model

We set out to formulate and solve a Consolidated Shipment Model in order to determine a potentially better redistribution plan. Our objective remains the same, but, our more complex consolidated shipment cost function requires us to formulate a more complex algebraic model. We provide a detailed formulation and model description in Appendix 3.

Determining consolidated shipment costs

To determine consolidated shipping costs, we set out to determine a piecewise linear cost function based on shipping weights along each origin-destination path. We consider each of the modes we used in our Initial Redistribution Model over a range of all possible weights. We then segment these weights by creating breakpoints at points where two cost functions either intersect or create a discontinuity, as shown in our example in figure A-1 in Appendix 3. For each segment, we choose the mode with the lowest cost over the entire segment as defined by the upper and lower weight breakpoints. Each line segment is the result of a linear approximation. After we identify each segment, we use the resulting cost function and associated breakpoints as model input data. As weight increases across the entire range of weights, the slopes of the cost functions decrease to form a generally concave function. To allow us to solve this problem, we assign binary variables and constraints that force the model to choose only one segment, or weight class, for each origin-destination pair.

Implementation

The Consolidated Shipments Model generates: 4,303 equations, 9,444 variables,

26,123 non-zero elements, and 1,071 discrete variables. Our GAMS mixed integer program model solves for an integer solution in 0.38 seconds using the XA solver (GAMS, 1997) on the same Dell 800 MHz personal computer we use to solve our Initial Redistribution Problem.

### Model output summary and comparison to initial model solution

Figure 5 compares the cost to fill all requirements for each model to the requirement to fill all deficiencies if the inventory is managed as independent sites. When comparing the totals, very little difference can be seen between the two models. However, the consolidated shipments model recommends a
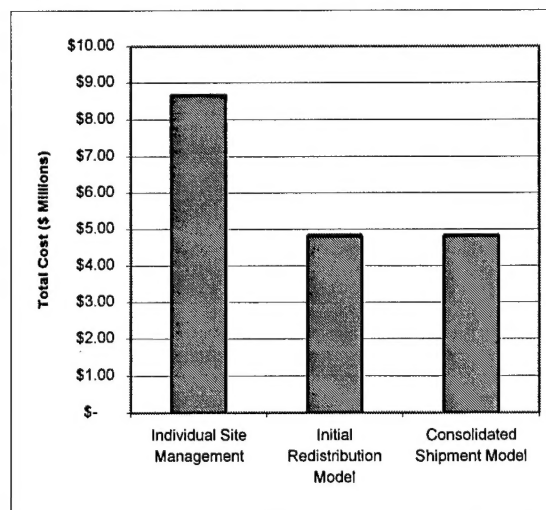
Figure 5. Comparing the total cost to fill all requirements for each model to the requirement to fill all deficiencies if the inventory is managed as independent sites.

redistribution plan to fill the 1003 deficiencies that could be effected with an associated shipping cost of $57 thousand and an additional $7 thousand of purchases. Compared to the $80 thousand to redistribute excesses from the Initial Redistribution Model plan, this represents a

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20% cost reduction. Figure 6 compares the costs for the two models in this context.
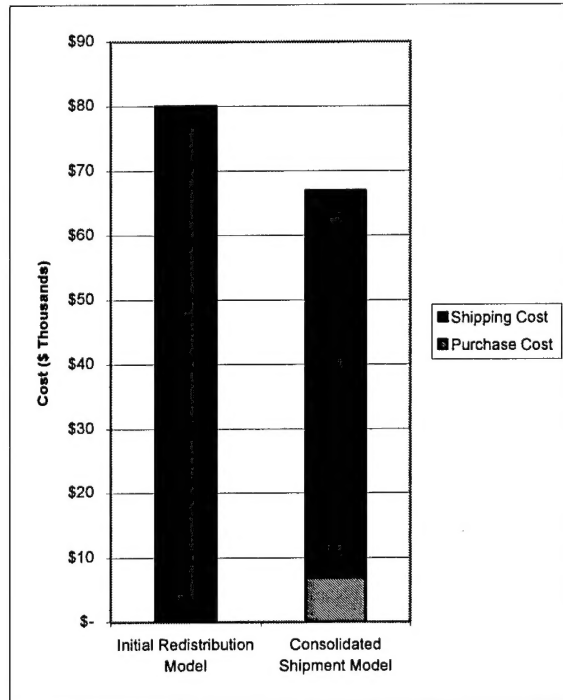
Figure 6. The Consolidated Shipments Model results in a 20% reduction in cost to fill all demand from sites with available stock. The model's recommendation to purchase some items instead of redistributing to fill all deficiencies reflects one solution where the Consolidated Shipments Model resulted in multiple optimal solutions.

Using Sensitivity Analysis for Economic and Operational Readiness Decisions

Our Consolidated Shipments Model provides a capability beyond just determining the minimum cost redistribution plan. A decision maker is also able to conduct sensitivity analysis from our model output to further improve their operational readiness at a minimum economic impact. Before we describe how sensitivity analysis can be used, we need to discuss the implications of not having a repairable part available at an inventory site when needed. In supply terms, this represents a backorder.

In the Marine Corps, each commander's principal goal is to be ready to perform specified missions within the constraints of their budget. Whenever a repairable part breaks, the associated vehicle or piece of equipment is not available to complete its mission. The sooner the broken part is replenished, the sooner the equipment can be returned to an operational status. Some inventory sites may have to backorder when the supply source cannot meet the required delivery date. In this case, redistributing on-hand stock from a site without excess may make sense. In general, the transport modes discussed in our model can deliver parts in a matter of days. To make this decision, a central manager needs to understand the estimated lead times, the variability associated with ordering new parts from a supply source, and the times for each type of mode to ship freight between sites. A redistribution to fill this critical part requirement will add to the total cost to fill all excesses because the shipping site will now have to order another part from a supply source. Our model allows the manager to determine the best shipping site, the one that would result in the lowest economic impact.

We identify the best candidates by looking at the reduced costs from our Consolidated Shipments model output. In the transportation problem, reduced cost is the cost per unit increase in the shipping decision variables (e.g. dollars per additional quantity shipped). In the context of our Consolidated Shipments Model, negative reduced cost is the cost difference between redistributing an item and its purchase cost. Negative reduced cost indicates that the deficient site had to purchase an item because no more excess was available at one, or more, of the other inventory sites. If other excess items were available, it would

have been shipped to the deficient site with the most negative reduced cost, resulting in the largest improvement in our objective function value. The Consolidated Shipments Model identifies candidates and calculates reduced costs, allowing decision makers to better understand the economic impact of getting a part when it's needed.

Summary and Conclusions

Our analysis shows three important lessons. First, we apply a basic operations research technique to this problem with significant results. We show that modifying our initial model to consider consolidated shipments lowers our total cost even more and is well worth considering in future modeling efforts. Additionally, the relatively short time to model and solve the problem could result in considerable manpower savings. Next, populating our model with data requires a large effort to ensure fidelity in our final solution. This is especially true when determining realistic cost estimates for both our per unit and consolidated shipments models. Our centralized managers must treat operating and accounting data as a critical resource. This will ensure they have accurate information to make important decisions. Finally, sensitivity analysis provides important insight for decision makers supported by these models and must be properly interpreted for their use.

APPENDIX 1: LIST OF ACRONYMS

AMC	Air Mobility Command
CONUS	Continental United States
DHL	DHL, International Worldwide Express
FEDEX	Federal Express
MTMC	Military Traffic Management Command
LTL	less than truckload
TL	truckload
WWX	Worldwide Express

APPENDIX 2: INITIAL REDISTRIBUTION MODEL FORMULATION

Sets and Indices

c commodities (National Stock Numbers or NIINs),

i sites with supply or demand (e.g. IMEF, BIC, HAWAII, ...),

j alias for i (refers to same set),

E set $\{\ (i, j\ ) : i$ could resupply j with a nonzero quantity of items $\}$,

Data

$d_{j,c}$ demand at site j for commodity c in number of items,

$s_{i,c}$ supply at site i of commodity c in number of items,

$tcost_{i,j,c}$ fixed cost for a shipment of commodity c from i to j,

$rcost_c$ replacement (purchase) cost per unit of commodity c;

Variables

$P_{j,c}$ qty of commodity c purchased locally by site j (number of items),

$X_{i,j,c}$ qty of commodity c shipped from i to j (number of items);

Formulation

Minimize

$$\sum_{i,j,k} t\cos t_{i,j,c} X_{i,j,c} + \sum_{j,c} rcost_c P_{j,c} \qquad [1]$$

Subject to:

$$\sum_{i:(i,j)\in E} X_{i,j,c} + P_{j,c} \geq d_{j,c} \quad \forall j,c \, ; \qquad [2]$$

$$\sum_{j:(i,j)\in E} X_{i,j,c} \leq s_{i,c} \quad \forall c, i:(i,j)\in E \, ; \qquad [3]$$

$$X_{i,j,c} = 0 \quad \forall c, (i,j) \notin E \, ; \qquad [4]$$

$$X, P \geq 0 \text{ and integer.} \qquad [5]$$

Data Description and Explanation

Our client provided the supply data, demand data, and replacement costs. Our calculations of unit transportation costs for each commodity between origin and destination sites is explained in detail during our explanation of the Initial Redistribution Model.

The Objective Function

The objective function [1] determines the total cost to fill all reparable part deficiencies by adding two terms that each make up linear combinations. The first term takes the sum of items redistributed between sites multiplied by the appropriate shiping costs. The second term takes the sum of items purchased multiplied by their respective purchase price.

Constraints

Constraints [2] requires that the total number of items shipped and purchased meet all demand at deficient sites. Constraints [3] limit the total quantity of each commodity shipped from each site to all others to at most the excess amount of that commodity at the site. Constraints [4] ensure no redistributions occur from sites where excesses are not available and to sites where deficiencies are not present. Constraints [5] sets the variables as integer and ensures standard non-negativity.

APPENDIX 3: CONSOLIDATED SHIPMENT MODEL FORMULATION

Sets and Indices

c commodities (National Stock Numbers or NIINs),

i sites with supply or demand (e.g. IMEF, BIC, HAWAII, ...),

j alias for i (refers to same set),

k weight classes, $k = 1, 2, ...,$ up to max number for any edge (i, j),

E set $\{ (i, j) : i$ could resupply j with a nonzero quantity of items $\}$,

Data

$brk_{i,j,k}$ upper bound of weight class k when shipping from i to j, in pounds,

$d_{j,c}$ demand at site j for commodity c in number of items,

$fixc_{i,j,k}$ fixed cost for a shipment from i to j in weight class k,

$s_{i,c}$ supply at site i of commodity c in number of items,

$rcost_c$ replacement (purchase) cost per unit of commodity c,

$varc_{i,j,k}$ variable cost per pound shipped from i to j in weight class k,

wt_c packaged weight per unit of commodity c, in pounds;

Variables

$P_{j,c}$ quantity of commodity c purchased locally by site j (number of items),

$W_{i,j,k}$ weight of all commodities shipped from i to j in class k,

$X_{i,j,c}$ quantity of commodity c shipped from i to j (number of items),

$Z_{i,j,k}$ binary variables, set if the total weight shipped from i to j is in class k;

Formulation

Minimize

$$\sum_{i,j,k}\left\{fixc_{i,j,k}Z_{i,j,k} + varc_{i,j,k}W_{i,j,k}\right\} + \sum_{j,c}rcost_c P_{j,c} \qquad [1]$$

Subject to:

$$\sum_{i:(i,j)\in E}X_{i,j,c} + P_{j,c} \geq d_{j,c} \quad \forall\, j,c\,; \qquad [2]$$

$$\sum_{j:(i,j)\in E}X_{i,j,c} \leq s_{i,c} \quad \forall\, c, i:(i,j)\in E\,; \qquad [3]$$

$$\sum_{k}W_{i,j,k} = \sum_{c}wt_c X_{i,j,c} \quad \forall\,(i,j)\in E\,; \qquad [4]$$

$$0 \leq W_{i,j,k} \leq brk_{i,j,k}Z_{i,j,k} \quad \forall\,(i,j)\in E, k=1\,; \qquad [5]$$

$$brk_{i,j,k-1}Z_{i,j,k} \leq W_{i,j,k} \leq brk_{i,j,k}Z_{i,j,k} \quad \forall\,(i,j)\in E, k>1\,; \qquad [6]$$

$$\sum_{k}Z_{i,j,k} = 1 \quad \forall\,(i,j)\in E\,; \qquad [7]$$

$$Z_{i,j,k} = 0 \quad \forall\, k,(i,j)\notin E\,; \qquad [8]$$

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$$X_{i,j,c} = 0 \ \forall \, c, (i, j) \notin E \,; \tag{9}$$

$$W_{i,j,k} = 0 \ \forall \, k, (i, j) \notin E \,; \tag{10}$$

$$X, P \geq 0 \text{ and integer}; W \geq 0; Z \text{ binary.} \tag{11}$$

Data Description and Explanation

The data used in this model are described in the previous discussion of the data used for the Initial Redistribution Model. Data used to determine consolidated shipment costs are discussed in detail below. The sources for these data remain the same. Figure A-1 represents a transportation cost function based on multiple modes for one origin-destination pair. The horizontal axis shows the weight shipped. The vertical axis represents the transportation cost for each mode (eg. TL, LTL, FedEx, WWX, etc.) and

associated weight. Each line segment has both fixed and variable costs for one mode. The breakpoints along the x-axis separate weight classes (k). These occur at points where the line segments intersect or there is a discontinuity. The weight limits for the modes determine the weight classes. For each weight class, we chose the one with the minimum cost. The resulting transportation cost function includes the fixed and variable costs of the mode with the cheapest mode for each weight class. This is done for each origin-destination pair.

x, shipping weight on path (i, j)
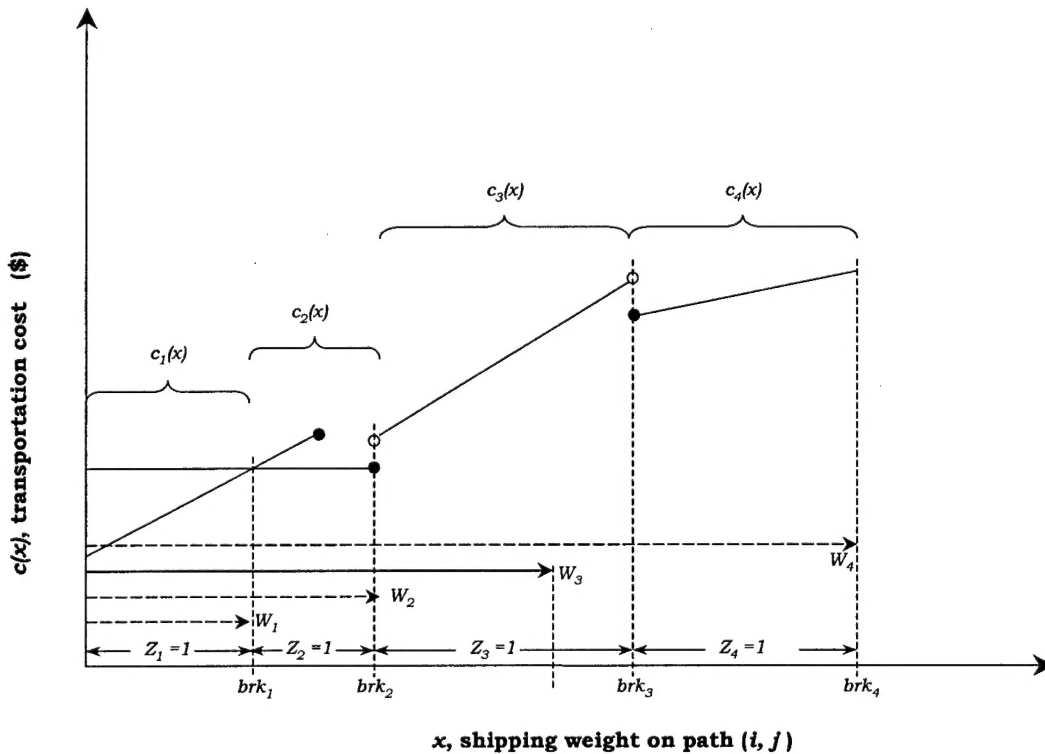
Figure A-1. Definition of variables in the context of a single edge (i, j). The weight classes, defined by the ranges shown for the binary variables Z, are bounded above by breakpoints provided in input data. There are four cost functions shown, and the separation between them is determined by placement of breakpoints. Note that the breakpoint between classes 1 and 2 is determined by the intersection of two cost functions for two modes that serve that weight class, but the remaining breakpoints are determined by a change of mode or weight class within mode. The horizontal dashed line between breakpoints 2 and 3, in weight class 3, depicts a notional total shipment weight from i to j. In this case, only one total weight variable W will be basic, and that is W_3 -- all others are zero as required by constraints [6].

Objective Function

The objective function [1] expresses total transportation and purchase cost to satisfy deficiencies at each site.

Constraints

Constraints [2] require the quantity of inbound redistributed items plus locally-purchased items to meet or exceed demand for each commodity at each site.
Constraints [3] limit the total quantity of a commodity shipped from each site to at most the supply. Constraints [4] equate total shipment weight from site i to site j, over all *commodities*, to total shipment weight from i to j over all *weight classes*. Constraints [5], [6], and [7] together consolidate all items shipped from i to j into a single shipment and determine the weight class of the shipment (see Figure Appendix 4). Side constraints [8]-[10] eliminate unnecessary variables, and [11] are standard nonnegativity constraints.

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